

Quiz ID: WHK

Name: _____

Answers:

1. ☐
2. ☐
3. ☐
4. ☐
5. ☐
6. ☐
7. ☐
8. ☐
9. ☐
10. ☐

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MAT 399**Quiz 1****Fall 2025**

For complex matrix A , let the adjoint of A , written A^* , denote the conjugate transpose $A^* = \overline{A^T}$, where conjugate means take complex conjugate of each entry, and transpose is the usual linear algebra operation, and the order of transpose and conjugate does not matter. Also let X , Y , and Z denote the Pauli matrices: $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, and I the identity matrix: $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, and let H be the Hadamard matrix: $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$.

1. Simplify and find the length of the complex number: $e^{i\frac{\pi}{2}}(1+i)$

- a) $\frac{\sqrt{3}}{2}$ b) $\sqrt{2}$ c) $2\sqrt{2}$ d) 1 e) 2

2. Find the matrix $A = XY$ (for Pauli matrices X and Y). What is the adjoint of A ?

- a) $\begin{pmatrix} i & 1 \\ -1 & i \end{pmatrix}$ b) $\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$ c) $\begin{pmatrix} -i & i \\ 1 & -1 \end{pmatrix}$ d) $\begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$ e) $\begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}$

3. Same matrix A as in previous question. What is A^2 ? (Let I be the identity matrix.)

- a) iI b) $2I$ c) I d) $-I$ e) $-iI$

4. Let R be the rotation matrix $R_{\frac{\pi}{4}}$ which rotates (counterclockwise) by $\frac{\pi}{4}$. What is R^4 ?

- a) iI b) $2I$ c) I d) $-iI$ e) $-I$

5. Same matrix R as in previous question. What is XR^2 ?

- a) I b) $-Z$ c) Z d) $-I$ e) Y

6. Let A be the complex matrix: HZH . Find the adjoint of A .

- a) I b) X c) H d) Y e) Z

7. Let A be the complex matrix: HXH . Find the adjoint of A . (Hint: use the previous question and multiply both sides on the left by H , then both sides on the right by H .)

- a) $-I$ b) X c) I d) Y e) Z

8. Find the matrix product XYZ :

- a) iI b) $2I$ c) $-iI$ d) I e) $-I$

9. Let $\mathbb{H}_1$ be the single qubit space with computational basis $\{|0\rangle, |1\rangle\}$. Let X be the Pauli matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Find the output of $X(2|0\rangle - 3|1\rangle)$ in the computational basis.

- a) $2|0\rangle - 3|1\rangle$ b) $3|0\rangle - 2|1\rangle$ c) $-3|0\rangle + 2|1\rangle$ d) $-|1\rangle$ e) $-|0\rangle$

10. Let $|00\rangle, |01\rangle, |10\rangle$, and $|11\rangle$ be the computational basis of $\mathbb{H}_2$, the 2 qubit space, which is the tensor product space of $\mathbb{H}_1$ with itself: $\mathbb{H}_2 = \mathbb{H}_1 \otimes \mathbb{H}_1$. Find the value of the tensor product $(|0\rangle + |1\rangle) \otimes (|0\rangle - |1\rangle)$ and write it as a coordinate vector with respect to the computational basis: $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.

- a) $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ b) $\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}$ c) $\begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$ d) $\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$ e) $\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$