MAT 399 **Quiz 1** **Fall 2025**

Quiz ID: ZFM Name: ____________________

Answers:

1. ☐
2. ☐
3. ☐
4. ☐
5. ☐
6. ☐
7. ☐
8. ☐
9. ☐
10. ☐

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For complex matrix A, let the adjoint of A, written A^*, denote the conjugate transpose $A^* = \overline{A^T}$, where conjugate means take complex conjugate of each entry, and transpose is the usual linear algebra operation, and the order of transpose and conjugate does not matter. Also let X, Y, and Z denote the Pauli matrices: $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, and I the identity matrix: $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, and let H be the Hadamard matrix: $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$.

1. Simplify and find the length of the complex number: $e^{i\frac{\pi}{2}}(1+i)$

   a) $\frac{\sqrt{3}}{2}$　　b) $\sqrt{2}$　　c) 2　　d) 1　　e) $2\sqrt{2}$

2. Find the matrix $A = XY$ (for Pauli matrices X and Y). What is the adjoint of A?

   a) $\begin{pmatrix} i & 1 \\ -1 & i \end{pmatrix}$　　b) $\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$　　c) $\begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}$　　d) $\begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$　　e) $\begin{pmatrix} -i & i \\ 1 & -1 \end{pmatrix}$

3. Same matrix A as in previous question. What is A^2? (Let I be the identity matrix.)

   a) iI　　b) $2I$　　c) $-iI$　　d) $-I$　　e) I

4. Let R be the rotation matrix $R_{\frac{\pi}{4}}$ which rotates (counterclockwise) by $\frac{\pi}{4}$. What is R^4?

   a) iI　　b) $2I$　　c) $-I$　　d) $-iI$　　e) I

5. Same matrix R as in previous question. What is XR^2?

   a) I　　b) $-Z$　　c) Y　　d) $-I$　　e) Z

6. Let A be the complex matrix: HZH. Find the adjoint of A.

a) I b) X c) Z d) Y e) H

7. Let A be the complex matrix: HXH. Find the adjoint of A. (Hint: use the previous question and multiply both sides on the left by H, then both sides on the right by H.)

a) $-I$ b) X c) Z d) Y e) I

8. Find the matrix product XYZ:

a) iI b) $2I$ c) $-I$ d) I e) $-iI$

9. Let $\mathbb{H}_1$ be the single qubit space with computational basis $\{|0\rangle, |1\rangle\}$. Let X be the Pauli matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Find the output of $X(2|0\rangle - 3|1\rangle)$ in the computational basis.

a) $2|0\rangle - 3|1\rangle$ b) $3|0\rangle - 2|1\rangle$ c) $-|0\rangle$ d) $-|1\rangle$ e) $-3|0\rangle + 2|1\rangle$

10. Let $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$ be the computational basis of $\mathbb{H}_2$, the 2 qubit space, which is the tensor product space of $\mathbb{H}_1$ with itself: $\mathbb{H}_2 = \mathbb{H}_1 \otimes \mathbb{H}_1$. Find the value of the tensor product $(|0\rangle + |1\rangle) \otimes (|0\rangle - |1\rangle)$ and write it as a coordinate vector with respect to the computational basis: $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.

a) $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ b) $\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}$ c) $\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ d) $\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$ e) $\begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$