

MAT 399**Quiz 1****Fall 2025**

For complex matrix A , let the adjoint of A , written A^* , denote the conjugate transpose $A^* = \overline{A^T}$, where conjugate means take complex conjugate of each entry, and transpose is the usual linear algebra operation, and the order of transpose and conjugate does not matter. Also let X , Y , and Z denote the Pauli matrices: $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, and I the identity matrix: $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, and let H be the Hadamard matrix: $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$.

1. Simplify and find the length of the complex number: $e^{i\frac{\pi}{2}}(1+i)$

a) 1 b) 2 c) $\frac{\sqrt{3}}{2}$ d) $2\sqrt{2}$ e) $\sqrt{2}$

Correct Answer: $\sqrt{2}$

2. Find the matrix $A = XY$ (for Pauli matrices X and Y). What is the adjoint of A ?

a) $\begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$ b) $\begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}$ c) $\begin{pmatrix} i & 1 \\ -1 & i \end{pmatrix}$ d) $\begin{pmatrix} -i & i \\ 1 & -1 \end{pmatrix}$ e) $\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$

Correct Answer: $\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$

3. Same matrix A as in previous question. What is A^2 ? (Let I be the identity matrix.)

a) $-I$ b) $-iI$ c) iI d) I e) $2I$

Correct Answer: $-I$

4. Let R be the rotation matrix $R_{\frac{\pi}{4}}$ which rotates (counterclockwise) by $\frac{\pi}{4}$. What is R^4 ?

a) $-iI$ b) $-I$ c) iI d) I e) $2I$

Correct Answer: $-I$

5. Same matrix R as in previous question. What is XR^2 ?

a) $-I$ b) Y c) I d) Z e) $-Z$

Correct Answer: Z

6. Let A be the complex matrix: HZH . Find the adjoint of A .

a) Y b) Z c) I d) H e) X

Correct Answer: X

7. Let A be the complex matrix: HXH . Find the adjoint of A . (Hint: use the previous question and multiply both sides on the left by H , then both sides on the right by H .)

a) Y b) Z c) $-I$ d) I e) X

Correct Answer: Z

8. Find the matrix product XYZ :

a) I b) $-I$ c) iI d) $-iI$ e) $2I$

Correct Answer: iI

9. Let \mathbb{H}_1 be the single qubit space with computational basis $\{|0\rangle, |1\rangle\}$. Let X be the Pauli matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Find the output of $X(2|0\rangle - 3|1\rangle)$ in the computational basis.

a) $-|1\rangle$ b) $-|0\rangle$ c) $2|0\rangle - 3|1\rangle$ d) $-3|0\rangle + 2|1\rangle$ e) $3|0\rangle - 2|1\rangle$

Correct Answer: $-3|0\rangle + 2|1\rangle$

10. Let $|00\rangle, |01\rangle, |10\rangle$, and $|11\rangle$ be the computational basis of \mathbb{H}_2 , the 2 qubit space, which is the tensor product space of \mathbb{H}_1 with itself: $\mathbb{H}_2 = \mathbb{H}_1 \otimes \mathbb{H}_1$. Find the value of the tensor product $(|0\rangle + |1\rangle) \otimes (|0\rangle - |1\rangle)$ and write it as a coordinate vector with respect to the computational basis: $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$.

a) $\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$ b) $\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ c) $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ d) $\begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$ e) $\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}$

Correct Answer: $\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$