Test ID: TESTID Name: _____

MAT 399 Quiz 2 Fall 2025

1. Consider the quantum circuit which applies a Hadamard gate to the first qubit followed by a standard CNOT gate with first qubit control and second qubit target such that the second qubit is flipped only when the first qubit equals 1. If A is the 4×4 matrix of this circuit (with respect to the computational basis), which of the following is a scalar multiple of the third column?

a) $\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ b) $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ c) $\begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$ d) $\begin{pmatrix} -1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$ e) $\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$

Correct Answer: $\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$

2. Same circuit as in the previous problem. If both input qubits are set to $|0\rangle$, what is the output in the computational basis?

a) $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ b) $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$ c) $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ d) $\frac{1}{\sqrt{2}}(|01\rangle - |11\rangle)$ e) $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

Correct Answer: $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

3. Same circuit as in the previous problem. If both input qubits are set to $|0\rangle$, and the final output is measured on both wires in the computational basis, what is the probability that the measurement will yield the basis state $|00\rangle$?

a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\frac{1}{4}$ d) 0 e) 1

Correct Answer: $\frac{1}{2}$

4. Same circuit as in the previous problem. If both input qubits are set to $|0\rangle$, and the final output is measured on both wires in the computational basis, what is the probability that the measurement will yield the basis state $|10\rangle$?

a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{2}}$ c) $\frac{1}{4}$ d) 0 e) 1

Correct Answer: 0

5. Let $f : \{0,1\}^2 \longrightarrow \{0,1\}$ be a Deutsch-Jozsa oracle function which is obtained from a standard CNOT circuit acting on the computational basis of $\mathbb{H}_2 = \mathbb{H}_1 \otimes \mathbb{H}_1$ by using input string ab converted to input $|a\rangle\,|b\rangle$ and using the output on the second qubit wire $|y\rangle$ so that $f(ab) = y$. True or False:

i) f is balanced ii) $f(00) = 1$ iii) $f(11) = 0$

a) TTT b) FTT c) TTF d) TFF e) TFT

Correct Answer: TFT

6. Let $f : \{0,1\}^2 \longrightarrow \{0,1\}$ be a Deutsch-Jozsa oracle function (so f is assumed to be either constant or balanced) with the property that $f(00) = 1$ and $f(11) = 0$. Also suppose a quantum circuit implements f with three wires (as done in class). So the first two wires will accept a bit string s of length two converted to two kets (left to right becomes top to bottom on the wires) and last (bottom wire) is always given input $|0\rangle$. The last wire produces the output ket $|y\rangle$ so that the function takes value $f(s) = y$. Also, suppose that the circuit consists of blocks with combinations of X and CCNOT gates (as done in class) and contains the following block: a single X on the first wire, followed by a CCNOT, followed by an X again on the first wire. This information implies which of the following?

   i) f is balanced ii) $f(01) = 1$ iii) $f(10) = 0$

   a) TTT b) FTT c) TTF d) TFF e) TFT

   Correct Answer: TTT