

MAT 399**Quiz 3****Fall 2026**

1. Consider the quantum circuit: which can be used to implement a function $f : \{0, 1\}^2 \rightarrow \{0, 1\}$, by the following rule: given any bit string of length two $ab \in \{0, 1\}^2$ define $f(ab)$ to be the label of the final state of the qubit on the third wire. Determine the signature string s of f in the following way: $s = s_0s_1s_2s_3$ where

$$s_0 = f(00), s_1 = f(01), s_2 = f(10), s_3 = f(11).$$

Equivalently, $s_i = f(b(i))$, where $b(i)$ is the two-digit binary representation of the index i , for $i = 0, 1, 2, 3$. Choose the correct s :

- a) 0011 b) 1001 c) 0000 d) 1100 e) 1011

Correct Answer: 0011

2. Same circuit and function f as in the previous problem. True or False:
i) f is balanced, ii) f is constant, iii) f is neither balanced nor constant.

- a) TFF b) FTF c) FFT d) TTF e) FFF

Correct Answer: TFF

3. Same circuit and function f as in the previous problem. Suppose you run the circuit 4 times, once on each of the 4 inputs with $ab \in \{0, 1\}^2$. How many times will the **first** CCNOT gate be triggered to act as a NOT gate on the third wire?

- a) 0 b) 2 c) 3 d) 1 e) 4

Correct Answer: 1

4. Same circuit and function f as in the previous problem. Suppose you run the circuit 4 times, once on each of the 4 inputs. How many times will the **second** CCNOT gate be triggered to act as a NOT gate on the third wire?

- a) 2 b) 1 c) 0 d) 4 e) 3

Correct Answer: 1

5. Same circuit and function f as in the previous problem. Suppose you run the circuit 4 times, once on each of the 4 inputs. How many times will **both** CCNOT gates be triggered to act as NOT gates on the third wire?

- a) 1 b) 2 c) 0 d) 3 e) 4

Correct Answer: 0