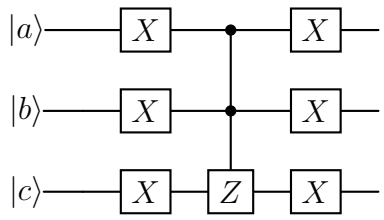
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MAT 399 Quiz 6 Fall 2026

1. Consider the quantum circuit:



What is the final state if $|a\rangle = |b\rangle = |c\rangle = |0\rangle$, so input is $|000\rangle$?

a) $|110\rangle$ b) $|001\rangle$ c) $-|111\rangle$ d) $-|000\rangle$ e) $-|001\rangle$

Correct Answer: $-|000\rangle$

2. Let V be the subspace of $\mathbb{H}_3$ consisting of all state vectors orthogonal to $|000\rangle$ and let $|\psi\rangle \in V$:

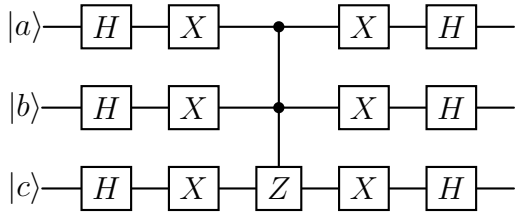
$$|\psi\rangle = \alpha_1 |001\rangle + \alpha_2 |010\rangle + \alpha_3 |011\rangle + \alpha_4 |100\rangle + \alpha_5 |101\rangle + \alpha_6 |110\rangle + \alpha_7 |111\rangle .$$

What is the final state if $|\psi\rangle$ is input into the same circuit as in the previous question?

a) $|111\rangle$ b) $|000\rangle$ c) $-|\psi\rangle$ d) $|\psi\rangle$ e) $-|000\rangle$

Correct Answer: $|\psi\rangle$

3. Consider the quantum circuit U:



Let $|s\rangle = H^{\otimes 3}|000\rangle$. What is the final state if the input is $|s\rangle$?

a) $-|s\rangle$ b) $|s\rangle$ c) $-|000\rangle$ d) $|000\rangle$ e) $-|111\rangle$

Correct Answer: $-|s\rangle$

4. Suppose $|\phi\rangle$ is orthogonal to $|s\rangle$ in $\mathbb{H}_3$. What is the final state if the input to the circuit U is $|\phi\rangle$ where U is the same circuit as in the previous problem. (Hint: the whole circuit U can be represented by a unitary transformation which preserves angles between vectors.)

a) $|\phi\rangle$ b) $-|\phi\rangle$ c) $-|000\rangle$ d) $|000\rangle$ e) $-|111\rangle$

Correct Answer: $|\phi\rangle$

5. Same circuit U and state $|s\rangle$ as in the previous problem. Let $U_s = -U$, and let I be the identity matrix of the same size as U. Suppose $|\psi\rangle$ is any state in $\mathbb{H}_3$, and P_s is the orthogonal projection onto $|s\rangle$. Which operator has the same effect on $|\psi\rangle$ as the operator $2P_s$?

a) $U_s + I$ b) $U_s - I$ c) U_s d) $U_s^2 - I$ e) I

Correct Answer: $U_s + I$