

MAT 399 Homework 5 – Fall 2025

Due Date: Wednesday, Dec 3

Assume equation (6.3.21) and also that $\omega \in [0, 1]$. In class we showed that if δ is defined by $N\omega = x + N\delta$ then under the extra assumption $\omega \in [0, 1]$ we have that $\delta = \Delta$. In all exercises from this section you can assume the above. So if I ask you to verify statement (x.x.x) this means to verify it under the above assumptions.

1. Show that equation (6.3.22) is now true.
2. Verify (6.3.23) by indicating where the Lemmas are used A.5.3 and A.5.5.
3. Assume $\theta > 0$ for this and then next exercises. Which value is x' ? Is it $x - 1$ or $x + 1$?
4. Establish the equations and inequalities of (6.3.25).
5. Verify (6.3.26) for $f(\theta)$ about its minimum value.
6. Explain why the author claims that the operator U_c defined in Definition 6.4.2 is linear.
7. Explain why the author claims, in the proof that U_c is unitary, that it suffices to show that f_c is a bijection.
8. Exercise 7.1.8 (use properties and results from chapter 2 to justify your claims)
9. Exercise 7.1.10