MAT 399 Homework 6 – Fall 2026

Due Date: Thursday, Nov 5

1. Exercise 7.1.4 on page 257. Note: This requires the changes to the definitions of $|s_0\rangle$ and $|s_1\rangle$ given in class:

$$|s_0\rangle = \frac{1}{\sqrt{N-M}} \sum_{\mathbf{x}\in\{0,1\}^n, f(\mathbf{x})=0} |\mathbf{x}\rangle\,, \quad |s_1\rangle = \frac{1}{\sqrt{M}} \sum_{\mathbf{x}\in\{0,1\}^n, f(\mathbf{x})=1} |\mathbf{x}\rangle\,.$$

2. Exercise 7.1.8 on page 259.

3. Exercise 7.1.10 on page 259.

4. Exercise 7.1.13 on page 260.

5. Let L_θ be the line in $\mathbb{R}^2$ that makes angle θ with the positive x axis. and let F_θ be the reflection across L_θ.

   (a) Show that F_θ can be expressed as a product of three matrices using the rotation R_θ and its inverse, and the reflection F_0 across the x axis.

   (b) Write the product from the previous part with entries each using a single trig function.

   (c) Test your result from the previous part for $\theta = \pi/4$ and $\theta = \pi/8$.

   (d) Show that the product of two reflections $F_{\theta_1}F_{\theta_2}$, with different angles θ_1 and θ_2, is a rotation, using matrix multiplication.

   (e) Test your result from the previous part for $\theta_1 = \pi/3$ and $\theta_2 = \pi/6$.