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MAT 399/597/599 CS 393/593 **Midterm Exam** Fall 2026

1. A quantum state vector for a single qubit is given as

$$|\psi\rangle = \alpha_0 |0\rangle + \frac{1}{\sqrt{5}} |1\rangle .$$

Choose a valid value for the amplitude α_0.

a) $\frac{4}{\sqrt{5}}$ b) $\frac{1}{\sqrt{5}}$ c) $\frac{3}{\sqrt{5}}$ d) $\frac{1}{\sqrt{2}}$ e) $\frac{2}{\sqrt{5}}$

Correct Answer: $\frac{2}{\sqrt{5}}$

2. A quantum state vector for a single qubit is given as

$$|\psi\rangle = \alpha_0 |0\rangle + \left(\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} i \right) |1\rangle .$$

Choose a valid value for the amplitude α_0.

a) $\frac{\sqrt{2}}{6}$ b) $\frac{\sqrt{2}}{2}$ c) $\frac{\sqrt{6}}{4}$ d) $\frac{\sqrt{6}}{2}$ e) $\frac{\sqrt{2}}{4}$

Correct Answer: $\frac{\sqrt{2}}{2}$

3. Same vector $|\psi\rangle$ and choice of α_0 in the previous problem. Find a Bloch Sphere representation of $|\psi\rangle$ of the form $(1, \theta, \phi)$. What is ϕ?

a) $\pi/3$ b) $\pi/4$ c) $\pi/2$ d) $\pi/6$ e) $2\pi/3$

Correct Answer: $\pi/3$

4. Suppose a 2-qubit register consists only of one CNOT gate, with control as first qubit and target as second qubit. If the input is $|x_+\rangle |x_-\rangle$ find the output.

a) $|x_+\rangle |x_+\rangle$ b) $|x_-\rangle |x_+\rangle$ c) $|x_-\rangle |x_-\rangle$ d) $\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$ e) $\frac{1}{\sqrt{2}} (|10\rangle + |01\rangle)$

Correct Answer: $|x_-\rangle |x_-\rangle$

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5. Suppose a 2-qubit circuit has unitary matrix with respect to the computational basis:

$$U = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

If this circuit has input $|ab\rangle$, with ab a bit string of length 2, the output is equivalent to:

a) $X|a\rangle|b\rangle$ b) $X^b|a\rangle X^{1-a}|b\rangle$ c) $|a\rangle X^a|b\rangle$ d) $|a\rangle X^{1-a}|b\rangle$ e) $|a\rangle X^b|b\rangle$

Correct Answer: $|a\rangle X^{1-a}|b\rangle$

6. The matrix U in the previous question satisfies which of the following properties:
i) unitary, ii) Hermitian, iii) involution ?

a) i) only b) i) and ii) only c) all three d) ii) only e) i) and iii)

Correct Answer: all three

7. Find the matrix A corresponding to the sum of outer products:

$$|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 1|.$$

a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ c) $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ e) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

Correct Answer: $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

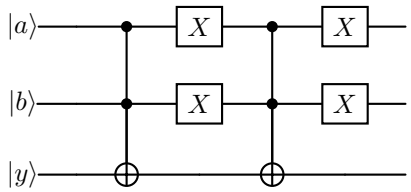
8. Same matrix A as in the previous problem. Choose an eigenvector of A.

a) $|0\rangle$ b) $|1\rangle$ c) $|x_+\rangle$ d) $|x_-\rangle$ e) $|0\rangle + i|1\rangle$

Correct Answer: $|0\rangle$

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9. Consider the quantum circuit U_f:



What is the final state if $|a\rangle = |b\rangle = |y\rangle = |0\rangle$, so input is $|000\rangle$?

a) $|100\rangle$ b) $|101\rangle$ c) $|111\rangle$ d) $|001\rangle$ e) $|011\rangle$

Correct Answer: $|001\rangle$

10. Same circuit and function f as in the previous problem. What is the function f, such that U_f computes f on the final state of the third qubit given that $y = 0$ and ab is a bit string of length 2? Determine the signature string s of f in the following way: $s = s_0 s_1 s_2 s_3$ where

$$s_0 = f(00),\ s_1 = f(01),\ s_2 = f(10),\ s_3 = f(11).$$

a) 1011 b) 1001 c) 0000 d) 1100 e) 1011

Correct Answer: 1001

11. Same circuit and function f as in the previous problem. Find the final state if $|a\rangle = |x_+\rangle$, $|b\rangle = |x_-\rangle$, and $|y\rangle = |0\rangle$. If we measure the final state in the computational basis of $\mathbb{H}_3$ what is the probability that we get value 0 (associated with bit string 000)?

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{1}{8}$ d) 1 e) 0

Correct Answer: 0

12. Same circuit and input and final state as in the previous problem. If we measure the final state in the computational basis of $\mathbb{H}_3$ what is the probability that we get value 7 (associated with the bit string 111)?

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{1}{8}$ d) 1 e) 0

Correct Answer: $\frac{1}{4}$

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13. Let A be the gate or matrix so that $A|0\rangle = \frac{1}{2}(\sqrt{3}|0\rangle + i|1\rangle)$ and $A|1\rangle = \frac{1}{2}(\sqrt{3}|1\rangle + i|0\rangle)$. Choose an eigenvalue of A:

a) $e^{i\pi/6}$ b) $e^{i5\pi/6}$ c) $e^{i\pi/4}$ d) $e^{i\pi/3}$ e) $e^{-i\pi/3}$

Correct Answer: $e^{i\pi/6}$

14. Same matrix A as in the previous question. Choose an eigenvector of A:

a) $|x_+\rangle$ b) $|0\rangle + i|1\rangle$ c) $|0\rangle$ d) $|1\rangle$ e) $|0\rangle + i\sqrt{3}|1\rangle$

Correct Answer: $|x_+\rangle$

15. Let R be the matrix:

$$R = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/3} \end{pmatrix}$$

and let $C(R)$ be the controlled R gate for a 2 qubit circuit with control on the first qubit and target on the second qubit, so that R is performed on the second qubit whenever there is a 1 in the first qubit. If this gate is performed on all four of the standard basis vectors of $\mathbb{H}_2$ how many of them will be unchanged?

a) 2 b) 1 c) 0 d) 4 e) 3

Correct Answer: 3

16. Same matrix R and gate $C(R)$ as in the previous problem. Suppose we input $|x_+\rangle$ on the first wire (control) and $|\psi\rangle$ on the second wire (target), where $|\psi\rangle$ is an eigenstate of R with eigenvalue $e^{i\pi/3}$, so we can write the input as $|x_+\rangle|\psi\rangle$. Also assume that the state $|\psi\rangle$ has length one as a complex vector in $\mathbb{C}^2$. The final output can be written as

$$\gamma_0 \frac{1}{\sqrt{2}}(\alpha_0|0\rangle + \alpha_1|1\rangle)(\beta_0|0\rangle + \beta_1|1\rangle).$$

Which coefficient assumes the value $e^{i\pi/3}$, while all the other coefficients are either 0 or 1?

a) α_0 b) α_1 c) β_0 d) γ_0 e) β_1

Correct Answer: α_1