Hamilton Cycle Splines			4-note chords are played with			3 and 7 up an octave and A110 = 0								
									y factor					
					(next)		Anchor Note		(melodic contour)		Transform		Folder	
Group	Bar	Chord	a, b, c, d	Type	Transition	pitch shift	cello	flute	cello	flute	cello	flute	cello	flute
1	1	A+M7	A,C#,E#,G#	[4,4,3]	a+	0,8,16,23	A110	Ab415	3	3	prime	prime	cello189/mel-s3	fluteA440/mel-s3
	2	Bbm7	Bb,Db,F,Ab	[3,4,3]	d-	1,8,16,23	Bb (+1)	Ab415	3	3	invers	invers	cello189/mel-s3	fluteA440/mel-s3
	3	G@7_1	Bb,Db,F,G	[3,4,2]	a-	1,8,16,22	Bb (+1)	G (-2)	3	3	retro	retro	cello189/mel-s3	fluteA440/mel-s3
	4	A7#5	A,C#,E#,G	[4,4,2]	d-	0,8,16,22	A110	G	3	3	ret-inv	ret-inv	cello189/mel-s3	fluteA440/mel-s3
2	5	F#mM7_1	A,C#,E#,F#	[4,4,1]	a+	0,8,16,21	C# (+4)	F# (-3)	3	3	prime	prime	cello189/mel-s3	fluteA440/mel-s3
	6	F#M7_1	A#,C#,E#,F#	[3,4,1]	b+	1,8,16,21	C#	F#	3	3	invers	invers	cello189/mel-s3	fluteA440/mel-s3
	7	F#+M7_1	A#,D,E#,F#	[4,3,1]	c-	1,8,17,21	D (+5)	F#	3	3	retro	retro	cello189/mel-s3	fluteA440/mel-s3
	8	F#7#5_1	A#,D,E,F#	[4,2,2]	c-	1,7,17,21	D	F#	3	3	ret-inv	ret-inv	cello189/mel-s3	fluteA440/mel-s3
3	9	EbmM7_2	Bb,D,Eb,Gb	[4,1,3]	d+	1,6,17,21	Eb (+6)	F#	3	3	prime	prime	cello189/mel-s3	fluteA440/mel-s3
	10	EbM7_2	Bb,D,Eb,G	[4,1,4]	a+	1,6,17,22	Eb	G (-2)	3	3	invers	invers	cello189/mel-s3	fluteA440/mel-s3
	11	Eb+M7_2	B,D,Eb,G	[3,1,4]	b-	2,6,17,22	Eb	G	3	3	retro	retro	cello189/mel-s3	fluteA440/mel-s3
	12	Eb7#5_2	B,Db,Eb,G	[2,2,4]	b-	2,6,16,22	Eb	G	3	3	ret-inv	ret-inv	cello189/mel-s3	fluteA440/mel-s3
4	13	CmM7_3	B,C,Eb,G	[1,3,4]	a-	2,6,15,22	C (+3)	Eb (-6)	2.5	2.5	prime	prime	cello189/mel-s2.5	fluteA440/mel-s2.5
	14	Cm7_3	Bb,C,Eb,G	[2,3,4]	d-	1,6,15,22	C	Eb	2.5	2.5	invers	invers	cello189/mel-s2.5	fluteA440/mel-s2.5
	15	C@7_3	Bb,C,Eb,Gb	[2,3,3]	a-	1,6,15,21	C	Eb	2.5	2.5	retro	retro	cello189/mel-s2.5	fluteA440/mel-s2.5
	16	Ao7	A,C,Eb,Gb	[3,3,3]	a-	0,6,15,21	C	Eb	2.5	2.5	ret-inv	ret-inv	cello189/mel-s2.5	fluteA440/mel-s2.5
5	17	Ab7	Ab,C,Eb,Gb	[4,3,3]	d+	-1,6,15,21	Ab (-1)	C (-9)	2	2	prime	prime	cello189/mel-s2.0	fluteA440/mel-s2.0
	18	AbM7	Ab,C,Eb,G	[4,3,4]	a+	-1,6,15,22	Ab	C	2	2	invers	invers	cello189/mel-s2.0	fluteA440/mel-s2.0
	19	A@7	A,C,Eb,G	[3,3,4]	b+	0,6,15,22	A (0)	C	2	2	retro	retro	cello189/mel-s2.0	fluteA440/mel-s2.0
	20	A7b5	A,C#,Eb,G	[4,2,4]	a+	0,6,16,22	A	C# (-8)	2	2	ret-inv	ret-inv	cello189/mel-s2.0	fluteA440/mel-s2.0
6	21	Eb7_2	Bb,Db,Eb,G	[3,2,4]	d-	1,6,16,22	C# (+4)	Eb (-6)	1.5	1.5	prime	prime	cello189/mel-s1.5	fluteA440/mel-s1.5
	22	Ebm7_2	Bb,Db,Eb,Gb	[3,2,3]	a-	1,6,16,21	C#	Eb	1.5	1.5	invers	invers	cello189/mel-s1.5	fluteA440/mel-s1.5
	23	Eb@7_2	A,Db,Eb,Gb	[4,2,3]	c+	0,6,16,21	C#	Eb	1.5	1.5	retro	retro	cello189/mel-s1.5	fluteA440/mel-s1.5
	24	F#m7_1	A,C#,E,F#	[4,3,2]	a+	0,7,16,21	C#	E (-5)	1.5	1.5	ret-inv	ret-inv	cello189/mel-s1.5	fluteA440/mel-s1.5
7	25	F#7_1	A#,C#,E,F#	[3,3,2]	b-	1,7,16,21	C# (+4)	E	1	1	prime	prime	cello189/mel-s1.0	fluteA440/mel-s1.0
	26	F#7b5_1	A#,C,E,F#	[2,4,2]	d+	1,7,15,21	C (+3)	E	1	1	invers	invers	cello189/mel-s1.0	fluteA440/mel-s1.0
	27	C7_3	A#,C,E,G	[2,4,3]	a+	1,7,15,22	C	E	1	1	retro	retro	cello189/mel-s1.0	fluteA440/mel-s1.0
	28	CM7_3	B,C,E,G	[1,4,3]	d+	2,7,15,22	C	E	1	1	ret-inv	ret-inv	cello189/mel-s1.0	fluteA440/mel-s1.0
8	29	C+M7_3	B,C,E,G#	[1,4,4]	a-	2,7,15,23	B (+2)	E (-5)	0.75	0.75	prime	prime	cello189/mel-s0.75	fluteA440/mel-s0.75
	30	C7#5_3	Bb,C,E,G#	[2,4,4]	a-	1,7,15,23	Bb (+1)	E	0.75	0.75	invers	invers	cello189/mel-s0.75	fluteA440/mel-s0.75
	31	AmM7	A,C,E,G#	[3,4,4]	a-	0,7,15,23	A (0)	E	0.75	0.75	retro	retro	cello189/mel-s0.75	fluteA440/mel-s0.75
	32	Ab+M	Ab,C,E,Ab	[4,4,4]	c+	-1,7,15,23	Ab (-1)	E	0.75	0.75	ret-inv	ret-inv	cello189/mel-s0.75	fluteA440/mel-s0.75

9	33	Fm_1	Ab,C,F,Ab	[4,5,3]	b-	-1,8,15,23	Ab (-1)	F (-4)	0.5	0.5	prime	prime	cello189/mel-s0.5	fluteA440/mel-s0.5
	34	Fo_1	Ab,B,F,Ab	[3,6,3]	c-	-1,8,14,23	Ab (-1)	F	0.5	0.5	invers	invers	cello189/mel-s0.5	fluteA440/mel-s0.5
	35	EM_1	G#,B,E,G#	[3,5,4]	c-	-1,7,14,23	G# (-1)	E (-5)	0.5	0.5	retro	retro	cello189/mel-s0.5	fluteA440/mel-s0.5
	36	G#m	G#,B,D#,G#	[3,4,5]	c-	-1,6,14,23	G# (-1)	D# (-6)	0.5	0.5	ret-inv	ret-inv	cello189/mel-s0.5	fluteA440/mel-s0.5
10	37	Abo	Ab,B,D,Ab	[3,3,6]	a-d-	-1,5,14,23	Ab (-1)	D (-7)	0.25	0.25	prime	prime	cello189/mel-s0.25	fluteA440/mel-s0.25
	38	G	G,B,D,G	[4,3,5]	a-d-	-2,5,14,22	G (-2)	D	0.25	0.25	invers	invers	cello189/mel-s0.25	fluteA440/mel-s0.25
	39	Bm_2	F#,B,D,F#	[5,3,4]	a-d-	-3,5,14,21	F# (-3)	D	0.25	0.25	retro	retro	cello189/mel-s0.25	fluteA440/mel-s0.25
	40	Bo_2	F,B,D,F	[6,3,3]	b-	-4,5,14,20	F (-4)	D	0.25	0.25	ret-inv	ret-inv	cello189/mel-s0.25	fluteA440/mel-s0.25
11	41	BbM_2	F,Bb,D,F	[5,4,3]	(a+)	-4,5,13,20	F (-4)	D (-7)	0	0	prime	prime	cello189/mel-s0.0	fluteA440/mel-s0.0
(here the cycle could return to type 1:)														
		F#+M7	Gb,Bb,D,F	[4,4,3]	etc									
Notes by column left to right:														
column														
1		group = 4 bar chunk for melodic transforms in order prime, inversion, retrograde, retrograde inversion												
2		bar = one chord arpeggiated then twice as block, 41 bars total for the Hamilton cycle through all types												
3		chord = seventh chord name, using underscore _n for inversion n, @ for half diminished, + for augmented												
4		a,b,c,d = notes of chord in standard form (within one octave), pitch low to high												
5		[x,y,z] = semitone separation type, x=b-a (# semitones between a and b), y=c-b, z=d-c												
6		transition = which note(s) to raise or lower by one semitone to reach the next chord in the sequence												
7		pitch shift = numeric value of notes with A110 = 0, counting up or down in semitones												
8		Anchor note (cello/flute) = one note in melodic fragment tied to this anchor note in chord,												
9		first note for prime and inversion, last for retrograde and retrograde inversion												
10		y factor (melodic) = scaled y axis for melodic contour, with max 3 chosen to give initial pitch spread of about one octave												
11		and scaling down in discrete chunks toward zero which means no change of y-value, so no change in pitch												
12		transform = prime (melodic fragment from spline cycle, scaled), inversion (negate y), retrograde (run backwards),												
13		retrograde inversion (run backwards and negate y)												
14		folder = directory (flute/cello) to find files for given melodic fragments												
15		(same)												
Other Notes:														
	1	Chords are played in Hamilton Cycle Splines with notes b and d up an octave, so in pitch increasing order: a,c,b,d												
	2	So the Dominant seventh chord in root position, with type [4,3,3] for example, is voiced: Ab,Eb,C,Gb												
	3	See [1] for details about the set of 41 [x,y,z] types and constraint-based systems of seventh chords.												
	4	The name is a pun on "cycle" which is used in the harmonic progression which follows a Hamilton Cycle (see [1]),												
		and in the melodic contours, which are generated using spline models of cycles extracted from audio samples, or Cycle Splines (see [2] and [3]).												

References: (at https://azrael.digipen.edu/research/)

[1] Constraint-Based Systems of Triads and Seventh Chords, and Parsimonious Voice-Leading, MCM 2019

[2] Spline modeling of audio signals and cycle interpolation, MCM2022

[3] Melodic Contour Generation with Spline Models of Cycles, MCM2024

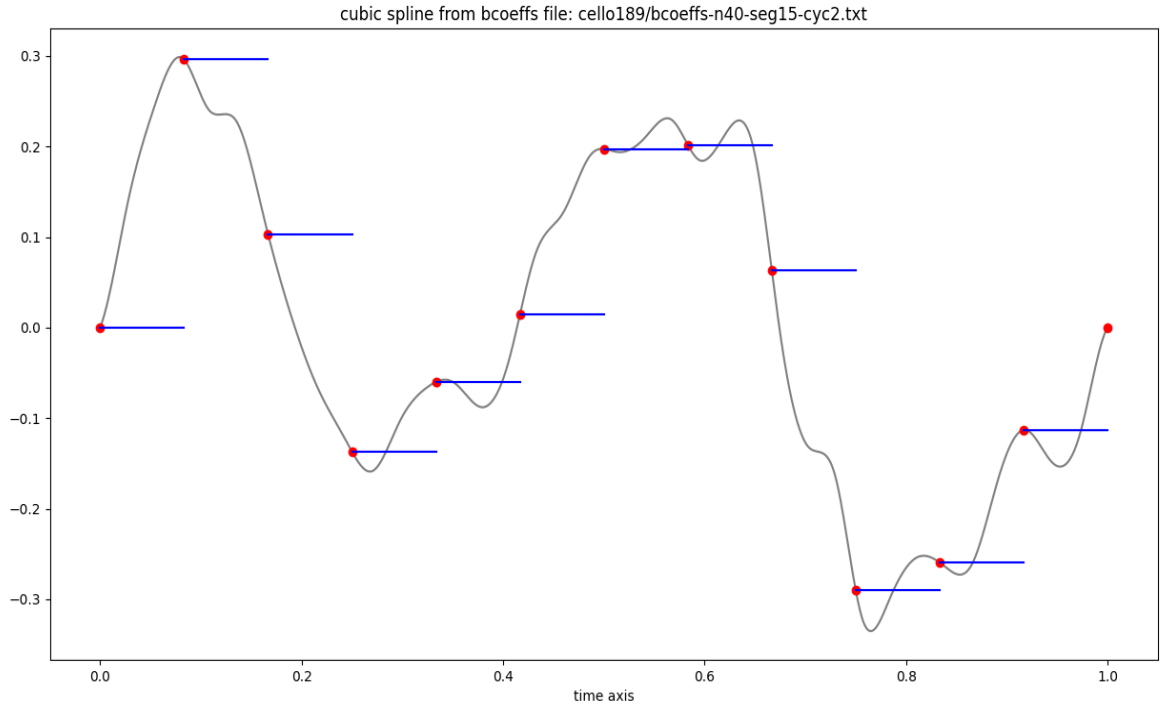
[4] TorchAudioSplines (github) https://github.com/mattjklassen/TorchAudioSplines

Notes on graphs of melodic contours:

1. The following two plots are outputs from matplot in python. Each plot represents one cycle from an audio sample of one note played on cello or flute.
2. For discussion of cycles based on zero crossings see [2] and for their use as melodic contours see [3].
3. These two contours are the only ones used in the composition Hamilton Cycle Splines.
4. The y factor scales the y axis before determining pitches with fundamental frequency f0 = F0 * 2^y, with some reference or starting frequency F0.
5. The cent value of the interval from F0 to F0 * 2^y is then simply 1200*y.
6. Since the cello max value is about 0.3 we get with scale factor 3 the max of about 0.9, or cent value 1200 * 0.9 = 1080, a slightly flat major seventh
7. Since the flute max value is about 0.275 we get with scale factor 3 the max of about 0.825, or cent value 1200 * 0.825 = 990, a slightly flat minor seventh
8. More precise cent values for the melodic fragments with y scale value 3 are given below after each plot.

Notes on video of Hamilton Cycle Splines playing back through Reaper:

1. The tracks in Reaper contain short wav file segments, with "cello-like" on top, chords in the middle, and "flute-like" at the bottom.
2. Bar numbers are at the top in small red dots, and chord symbols appear as labels at the top of each bar.
3. A "+1" or a "-1" appears in front of any voice which is about to change by one semitone.
4. In the first 3 groups (12 bars) there is no change to the pitch spread in the melodic parts, since all have y factor 3.
5. In the first 3 groups the melodic parts are shifted about, or transposed, which can be seen by noting the anchor notes.
6. In bar 13 the melodic fragments are slightly contracted in spread, with y factor 2.5.
7. Starting with bar 13, or group 4, each successive group has contracted pitch spread.



melody summary:

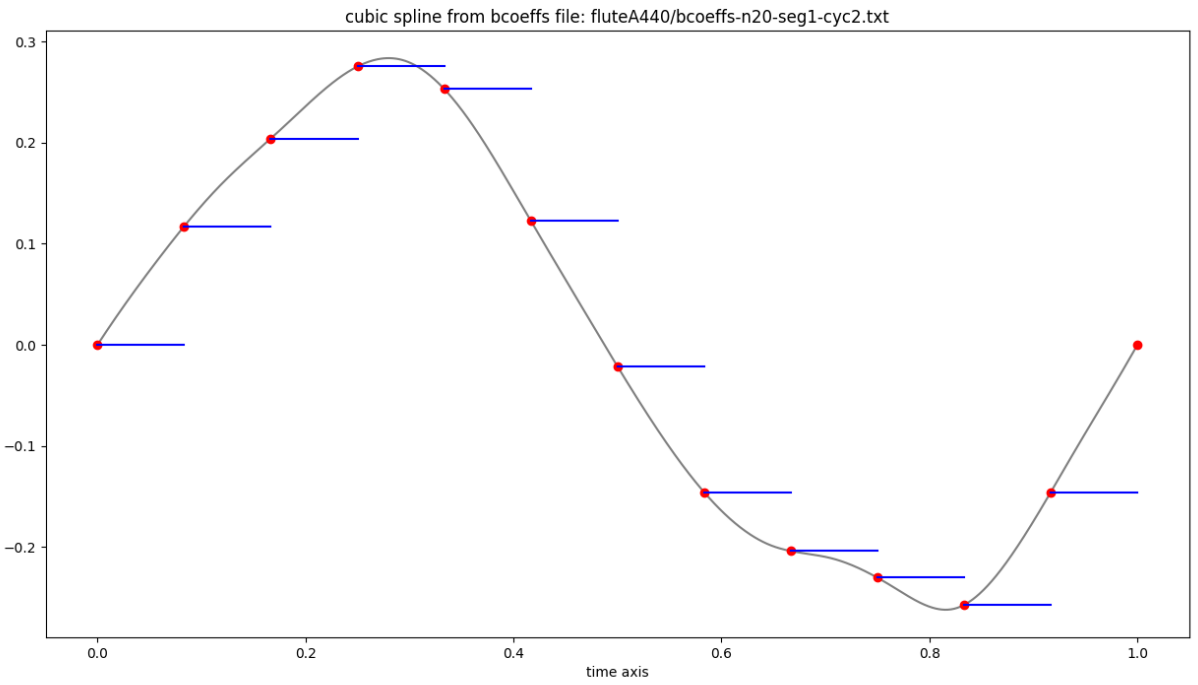
number of notes = 12, initial f0 = 110.0

first note duration = 0.125, total time in seconds = 1.5

prime sequence of intervals between notes as cent values:
1065, -693, -864, 277, 267, 658, 15, -498, -1272, 110, 527

prime sequence of intervals relative to initial f0 as 0:
0, 1065, 371, -493, -215, 51, 710, 726, 228, -1044, -933, -406

prime sequence of fundamental frequency f0 values:
110, 203, 136, 82, 97, 113, 165, 167, 125, 60, 64, 86



number of notes = 12, initial f0 = 440.0

first note duration = 0.125, total time in seconds = 1.5

prime sequence of intervals between notes as cent values:
422, 312, 256, -79, -470, -518, -448, -209, -95, -97, 401

prime sequence of intervals relative to initial f0 as 0:
0, 422, 735, 991, 911, 441, -76, -525, -734, -829, -926, -525

prime sequence of fundamental frequency f0 values:
440, 561, 672, 780, 745, 567, 420, 324, 287, 272, 257, 324